Exercises

Luxformel

Vectors and Matrices

Exercise 1

Consider the following vectors in $\mathbb{R}^7$:

$$u = (0.5, 0.4, 0.4, 0.5, 0.1, 0.4, 0.1), \quad v = (-1, -2, 1, -2, 3, 1, -5)$$

1. Check if u and v are unit vectors.

2. Calculate the dot product of the vectors u and v.

3. Are u and v orthogonal?

Exercise 2

Consider the following vectors in $\mathbb{R}^9$:

$$u = (1, 2, 5, 2, -3, 1, 2, 6, 2),$$

$$v = (-4, 3, -2, 2, 1, -3, 4, 1, -2)$$

$$w = (3, 3, -3, -1, 6, -1, 2, -5, -7)$$

1. Which pairs of these vectors are orthogonal?

2. Calculate the Euclidean norm of u.

3. Calculate the infinity norm of w.

Exercise 3

Consider the following matrices:

$$A = \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$

$$E = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad F = \begin{pmatrix} -2 & 1 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 4 & 0 & 0 \end{pmatrix}$$

1. Calculate, if possible:
   - $A + B$
   - $B - A$
   - $B + C$
   - AB
   - BA
   - BG
   - CE
   - EF
   - FE
2. Write the transposes of A and B and calculate their product. Which property can one observe?

Exercise 4

Consider the following matrices:

$$A = \begin{pmatrix} 2 & -2 \\ -3 & 1 \\ 5 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 4 & 4 \\ -2 & 3 & -7 \\ 2 & 5 & -7 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & -1 & 2 \\ -8 & 2 & -4 \\ 2 & 1 & -4 \end{pmatrix}$$

1. Compute $A^T B$ and $C + B$.

2. Which of the matrices A, B, C are full rank?

3. Calculate the Frobenius norm of C and the spectral norm of A.

4. Calculate the inverse of B.

Exercise 5

Consider the following matrices:

$$A = \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$

1. Calculate the determinants of the matrices A, B, and AB.

2. Calculate the determinants of the matrices C and D.

Exercise 6

Consider the following matrices:

$$A = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 4 & 5 \end{pmatrix}, \quad C = \begin{pmatrix} 6 & -9 \\ -4 & 6 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 6 & 2 \\ 0 & 1 & 0 \\ 3 & 0 & -5 \end{pmatrix}$$

Calculate, if possible, the inverses of the matrices A, B, C, and D.

Exercise 7

Consider the matrix:

$$A = \begin{pmatrix} 2 & 2 & 3 \\ -2 & 7 & 4 \\ -3 & -3 & -4 \\ -8 & 2 & 3 \end{pmatrix}$$

1. Add a column to A so that it is invertible.

2. Remove a row from A so that it is invertible.

3. Calculate AA^T. Is it invertible?

4. Calculate A^TA. Is it invertible?

Exercise 8

1. Calculate the inverse of the matrix $M = \begin{pmatrix} 3 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -4 & 5 \end{pmatrix}$.

2. Use this inverse to solve the linear system:

$$\begin{cases} 3x + 2y - z = 5 \\ x - y + z = 1 \\ 2x - 4y + 5z = -3 \end{cases}$$

Exercise 9

Solve the systems:

1.
$$\begin{cases} 2x + 3y + 5z = 2 \\ 7x + z = -1 \\ -2y + 2z = 3 \end{cases}$$

2.
$$\begin{cases} x + 2y - z = 2 \\ 2x + 5y + 4z = 3 \\ 3x + 7y + 4z = 1 \end{cases}$$