Exercises

Luxformel

Complex Numbers and Vector Space

Exercise 1: Complex Number Inverse

Suppose a and b are real numbers, not both 0. Find real numbers c and d such that

$$\frac{1}{a+bi} = c + di$$

Exercise 2: Cube Root of Unity

Show that

$$\frac{-1+\sqrt{3}i}{2}$$

is a cube root of 1 (meaning that its cube equals 1).

Exercise 3: Square Roots of i

Find two distinct square roots of i (i.e., find two different complex numbers z such that $z^2 = i$).

Exercise 4

Prove the following properties and name them:

1. Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in \mathbb{C}$.
2. Show that $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.
3. Show that $(\alpha\beta)\lambda = \alpha(\beta\lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.
4. Show that for every $\alpha \in \mathbb{C}$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha + \beta = 0$.
5. Show that for every $\alpha \in \mathbb{C}$ with $\alpha \neq 0$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha\beta = 1$.
6. Show that $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$ for all $\lambda, \alpha, \beta \in \mathbb{C}$.

Exercise 5: Vector Equation in $\mathbb{R}^4$

Find $x \in \mathbb{R}^4$ such that

$$(4, -3, 1, 7) + 2x = (5, 9, -6, 8).$$

Exercise 6: Non-Existence of Scalar Multiplication

Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

Exercise 7

Prove the following:

1. Show that $(x + y) + z = x + (y + z)$ for all $x, y, z \in \mathbb{F}^n$.
2. Show that $(ab)x = a(bx)$ for all $x \in \mathbb{F}^n$ and all $a, b \in \mathbb{F}$.
3. Show that $1x = x$ for all $x \in \mathbb{F}^n$, where 1 is the multiplicative identity in $\mathbb{F}$.
4. Show that $\lambda(x + y) = \lambda x + \lambda y$ for all $\lambda \in \mathbb{F}$ and all $x, y \in \mathbb{F}^n$.
5. Show that $(a + b)x = ax + bx$ for all $a, b \in \mathbb{F}$ and all $x \in \mathbb{F}^n$.